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EXIOPOL

A NEW ENVIRONMENTAL ACCOUNTING FRAMEWORK USING EXTERNALITY DATA AND INPUT-OUTPUT TOOLS FOR POLICY ANALYSIS

Technical Report on Basic Conceptions for Environmental Extensions of Input-Output Framework

Deliverable DIII.2.b-1 & DIII.3.b-1

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Preamble

EXIOPOL ("A New Environmental Accounting Framework Using Externality Data and Input-Output Tools for Policy Analysis") is an Integrated Project set up by FEEM and TNO with funding from the EU's 7th Framework Program. It runs between March 2007 and 2011. The main project set-up is in three content clusters, one on externalities modelling (Cluster II), one on a SUT/IO accounting framework with environmental extensions (Cluster III), and one on using the combined result in modelling for decision support (Cluster IV). Furthermore, one overarching cluster is dedicated to keeping the scope of this conceptually complex project focused (Cluster I), one is reserved for management (Cluster VI), and a final one is for dissemination of results (Cluster V).

This technical report combines two deliverables: DIII.2.b-1 and DIII3.b.1. As outlined in the "Draft Planning for the Next 18 Month (11 April 2008)" work in Cluster III has been reorganized slightly. A "new" workstream has been defined combining work packages WPIII.2.b and WPIII.3.b, both dealing with the environmental extensions.

The present report relates to the Environmental Extensions part of Cluster III. It focuses on the theoretical-mathematical conceptions of linking environmental extensions (EE) to the framework of Supply- and Use-Tables (SUT) as applied in Cluster III.

A detailed description of the environmental extension data is already provided in deliverable DIII.1.a-2 (Moll et al. 2007) which can be downloaded, as other documents, from the EXIOPOL website: http://www.feem-project.net/exiopol/index.php.

Executive Summary

This technical report relates to Cluster III and focuses on the basic conceptions for environmental extensions in an Input-Output framework such as the Supply-Use framework as utilised in EXIOPOL. Section 2 introduces into the framework of Supply- and Use Tables (SUT). A framework for linking environmental extensions to this SUT framework is presented in section 3. Starting from the existing monetary models, the mathematical-theoretical models for environmental extensions (EE- SUT) are developed in section 4. Annex 1 gives a numerical example to illustrate the different models.

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1 Introduction

The overall objective of WPIII.2.b and WPIII.3.b is to gather and process environmental data so as to build up the environmental extensions modules (EE) to the monetary EXIOPOL-framework of Supply- and Use Tables (SUT).

There are several conceptual possibilities to link environmental extensions to the overall EXIOPOL framework (see also deliverable DIII.1.a-2, Moll et al. 2007).

Basic conceptual choices related to EE-conceptions were taken particularly at the Leiden workshop of the Cluster III team:

- Supply-Use Tables (SUT) are to be used as the overall concept of the EXIOPOL IO-framework (=> EE-SUT)
- EE are to be linked to the SUT-framework in form of satellite accounts

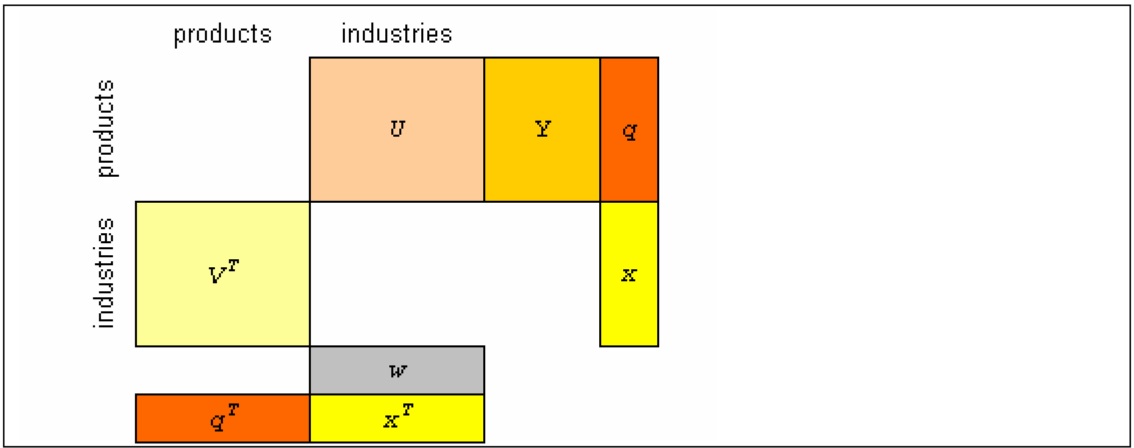
2 The SUT framework as applied in EXIOPOL Cluster III

Figure 1 shows a simplified scheme of the monetary (in basic prices) SUT-framework as applied by EXIOPOL (see also Table 2.3 in Tukker and Heijungs 2008 pp. 13-22). The scheme combines and integrates supply and use tables.

The original supply table – which needs to be rotated by 90° in such a combined SUT scheme – is mainly to be found along the first column of Figure 1. Here, $\boldsymbol{V^T}$ is the transposed central element of the supply table, sometimes referred to as production matrix. An element V_{ij} indicates the supply of product *i* from domestic production by industry *j*. The vector $\boldsymbol{m^T}$ is the transposed vector of imported products *i*. The vector $\boldsymbol{q^T}$ is the transposed total supply by products *i* (either domestically produced by industries or through imports). The vector $\boldsymbol{x}$ is the output by industries (domestic production).

The original use table is mainly to be found along the first row of the combined SUT-scheme. Here, the matrix $\boldsymbol{U}$ denotes the inter-industry part of the use table, the so-called intermediate consumption matrix. An element of U_{ij} indicates the use of product *i* by industry *j* (including imported intermediate goods). The matrix $\boldsymbol{Y}$ denotes the second part of the use matrix – the final demand matrix. It shows the use of product *i* by several components of final demand (e.g. consumption by private households and government, gross fixed capital formation, exports). Vector $\boldsymbol{q}$ shows the total of products used by industries and components of final demand. The vector $\boldsymbol{w}$ denotes the gross value added by industries (comprising several factor inputs such as wages, depreciation, surplus etc.). Finally, the transposed vector $\boldsymbol{x^T}$ shows all inputs (intermediate products and value-added) to industries.

Figure 1: General SUT-framework



There are two basic identities inherent to the SUT-scheme presented in Figure 1. First, the total use of products (q) is equal to the total supply of products (q^T). Secondly, the total input to industries (x^T) is equal to the total output of industries (x). For a mathematical formalisation of this, see Tukker and Heijungs 2008 pp. 15-16.

The SUT-scheme serves as the data-wise point of departure for economic model calculations. A possible application of such model calculation is e.g. to take the final use as the independent variable in order to analyse research questions such as e.g. what are the consequences of changes in final demand? For instance how much needs to be produced additionally in order to enable an increase in final demand for product *i*? Model calculations based on the SUT-scheme allow answering such kind of questions in terms of industry production outputs and/or products. Thereby, several assumptions are being made (see Tukker and Heijungs 2008 pp. 17-19).

3 Environmental Extensions (EE-SUT) - framework

Environmental Extensions (EE) are certain environmentally relevant parameters in physical units which are to be linked to the monetary SUT-scheme (shown in Figure 1) in order to enable integrated environmental and economic model calculations. One possible analytical objective may be to quantify environmental consequences of changes in final demand. An increase in final demand will cause – likewise the additional production and product output – also additional utilisation of environmental extensions.

The SUT-scheme as illustrated in Figure 1 is a balanced system as it is expressed solely in monetary units. The monetary input to industries equals the monetary output of industries and the product supply in monetary units equals the product use in monetary units. These identities are an important prerequisite for the mathematical Leontief-type models.

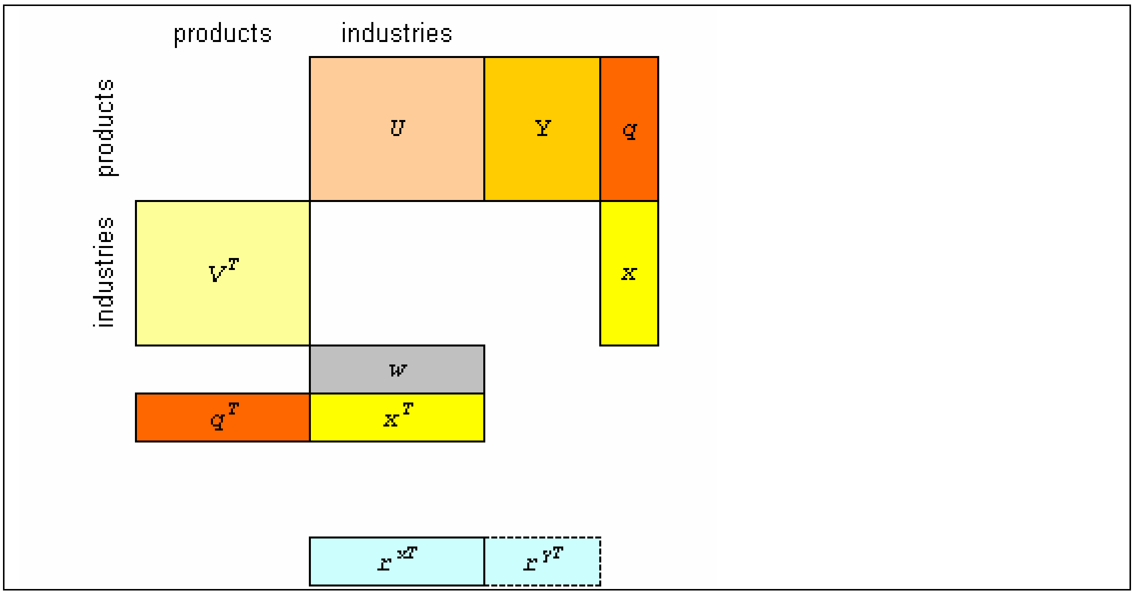
Hence, it is reasonable to arrange the environmental extension parameters external to the balanced monetary SUT-scheme. This is done via so-called satellite accounts. They form a simple possibility for adding environmental extension parameters (but also physical representations of factor inputs, such as e.g. labour in working hours or number of occupied persons) to the monetary SUT-scheme.

Satellite accounts are external vectors and/or matrices which are simply added to the monetary core SUT-scheme, arranged in a compatible way to the SUT column and row headings.

The most common way is to arrange the environmental extensions to the use table part, likewise the value added. Figure 2 shows how a single environmental extension (e.g. CO_2-emissions) vector can be externally added to the SUT-scheme. Actually, this environmental extension vector comprises two parts, one related to industries and another to the several components of final use[1]. The vector $\boldsymbol{r}^{x}$ shows the direct environmental factor input (e.g. CO_2-emissions) by industries j. The vector $\boldsymbol{r}^{y}$ shows the environmental factor input to components of final use, basically private households.

[1] Note that private households are usually the only final use component to which environmental extensions are assigned (e.g. direct CO_2-emissions by private households).

Figure 2: General SUT scheme extended by one single external environmental extension vector



Obviously, one can also add more than one environmental extension variable leading to matrices of environmental extensions ***R***.

4 Environmental Extensions (EE-SUT) - mathematics

The SUT-scheme (as shown in Figure 1) constitutes the data base from where several mathematical models can be derived (see Tukker and Heijungs 2008 pp. 15-19; ten Raa and Rueda-Cantuche 2007). This section outlines how environmental extensions can be linked to and integrated into these mathematical models.

The environmental extension vector is related to industries as shown in the scheme of Figure 2. A simple linear relationship is assumed between the production output x_j of a given industry j and its use of an environmental extension $r^x{}_j$ leading to the intensity $s^x{}_j$ for this particular industry j:

$$s^x{}_j = r^x{}_j / x_j \tag{1}$$

The intensity represents the amount of environmental extension per unit monetary production output. All elements of $s^x{}_j$ together form the vector of environmental intensities $\boldsymbol{s^x}$.

$$\boldsymbol{s^{xT}} = [\ s^x{}_1 \ \ s^x{}_2 \ \ s^x{}_3 \ \ \dots \ \ s^x{}_j\ ] \tag{2}$$

In matrix notation the derivation of the vector of environmental intensities by industries reads like:

$$\boldsymbol{s}^{\boldsymbol{xT}} = \boldsymbol{r}^{\boldsymbol{xT}} \cdot \hat{\boldsymbol{x}}^{-1} \tag{3}$$

with:

- $\boldsymbol{r^x}$: environmental extension vector by industries
- $\boldsymbol{x}$: production output by industries
- $\boldsymbol{s^x}$: vector of environmental intensities by industries

Note: the subscript T denotes the transposed of a vector or matrix; the hat symbol ^ denotes that the respective vector is diagonalized.

In addition to the vector of environmental intensities by industries, the various SUT-models also require knowing the environmental extension and respective vector of environmental intensities broken down by products. $r^q{}_j$ denotes the environmental extension by products and it represents the directly utilised environmental extension to all production processes producing product q_j.

Accordingly, $s^q{}_j$ denotes the environmental intensity of products q_j. We derive this vector of environmental intensities by products $\boldsymbol{s^q}$ through multiplying the inverse of the transposed production matrix $\boldsymbol{V^{-T}}$ from left with the environmental extension vector by industries $\boldsymbol{r^x}$:

$$s^q = V^T \cdot r^x \qquad (4)$$

Accordingly, the corresponding environmental extension vector by product can be obtained through multiplying the product output q with the previously obtained environmental intensity of products s^q:

$$r^{qT} = q^T \cdot \hat{s}^q \qquad (5)$$

with:

s^q : environmental intensities by products

V : production matrix

r^q : environmental factor input vector by products

q : product output

Supply and use tables (SUT) constitute the data base from which the data base for macroeconomic models can be derived in the form of symmetric input-output tables. Building on different assumptions, there are several basic models for the transformation of supply and use tables to symmetric input-output tables. The EXIOPOL-deliverable DIII.1.a-5 (Tukker and Heijungs 2008) pp. 15-19 presents several transformation models whereby a general distinction is made with regards to two technology assumptions:

- Industry-technology assumption (ITA) assumes that all industries have the same input structure (technology) regardless of the product they produce. For instance, assume the car manufacturing industry is producing cars as a principal output and electricity as a secondary output. Under the ITA, it is assumed that both products, i.e. the cars and the electricity, are produced with the same technology and respective recipe of inputs.
- Commodity-technology assumption (CPA) assumes that all products have the same input structure (technology) regardless of the industry that produces it. For instance, assuming again that the car manufacturing industry is producing both, cars and electricity. Under the CTA it is assumed that the latter electricity by the car industry is produced with the input structure (technology) as observed in the electricity industry.

It has to be noted, that the recently published *Eurostat Manual of Supply, Use and Input-Output Tables* (Eurostat 2008, pp. 295 ff.) suggests a distinction of transformation models which is different from that by Tukker and Heijungs. The Eurostat manual suggests a revised terminology which refers to the categories of technology and market shares.

Industry-technology assumption (ITA)

The system of equations related to the ITA – two equations in two unknowns – can be represented in the following supply-use block (eq.[16] in Tukker and Heijungs 2008):

$$\begin{pmatrix} q \\ x \end{pmatrix} = \begin{pmatrix} 0 & U \cdot \hat{x}^{-1} \\ V^T \cdot \hat{q}^{-1} & 0 \end{pmatrix} \begin{pmatrix} q \\ x \end{pmatrix} + \begin{pmatrix} y \\ 0 \end{pmatrix} \quad (6)$$

Solving this system for industry output gives the industry-by-industry requirement matrix, i.e. the central block in an industry-by-industry IOT. The Eurostat manual terms this *fixed products sales structure* industry by industry model (Eurostat 2008). The related Leontief-type model reads like following:

$$\boldsymbol{x} = (\boldsymbol{I}\text{-}\boldsymbol{A}^{\boldsymbol{x}})^{-1} \cdot \boldsymbol{y}^{\boldsymbol{x}} \quad (7)$$

where

$$\boldsymbol{A}^{x} = \boldsymbol{V}^{T} \cdot \hat{\boldsymbol{q}}^{-1} \cdot \boldsymbol{U} \cdot \hat{\boldsymbol{x}}^{-1} \quad (8)$$

and

$$\boldsymbol{y}^{x} = \boldsymbol{V}^{T} \cdot \hat{\boldsymbol{q}}^{-1} \cdot \boldsymbol{y} \quad (9)$$

The environmental extension is linked by adding $\boldsymbol{s}^{q}$, the environmental intensities by industries, to the right side of the equation:

$$\boldsymbol{R}^{x} = \hat{\boldsymbol{s}}^{x} \cdot (\boldsymbol{I} - \boldsymbol{A}^{x})^{-1} \cdot \hat{\boldsymbol{y}}^{x} \quad (10)$$

Each element R^{x}_{ij} of this resulting physical industry by industry matrix shows, how much of the environmental extension is used by industry i to produce intermediate goods for industry j in order to enable the latter to produce its goods for final demand. The row totals show the direct use of the respective environmental parameter by industries i, i.e. the original environmental extension vector by industries ($\boldsymbol{r}^{x}$).

The column totals show the direct and indirect amount of respective environmental extension parameter activated by the final demand of industry outputs j.

Solving the above ITA system for product outputs gives the product-by-product requirements matrix, the central block of a product-by-products IOT. The related Leontief-type model is:

$$\boldsymbol{q} = (\boldsymbol{I}\text{-}\boldsymbol{A}^{\boldsymbol{q}})^{-1} \cdot \boldsymbol{y} \quad (11)$$

where

$$\boldsymbol{A}^{q} = \boldsymbol{U} \cdot \hat{\boldsymbol{x}}^{-1} \cdot \boldsymbol{V}^{T} \cdot \hat{\boldsymbol{q}}^{-1} \quad (12)$$

Again, the environmental extension is integrated by adding the vector of environmental intensities to the model equation; however, this time the intensities-vector is by products, i.e. $\boldsymbol{s^q}$:

$$\boldsymbol{R^q = \hat{s}^q \cdot (I - A^q)^{-1} \cdot \hat{y}} \tag{13}$$

Each element $\boldsymbol{R^q}_{ij}$ of this product by product matrix shows how much of an environmental extension is directly used for the production of product *i* to be delivered as intermediate good for the production of final demand of product *j*. The row totals show the direct use of an environmental extension needed for total output of product *i*, i.e. the environmental extension vector broken down by products ($\boldsymbol{r^q}$).

The column totals show the direct and indirect amount of an environmental extension activated by the final demand of product *j*.

Finally, a mixed system can be derived that takes the final demand in products and returns the industry output which probably constitutes the most typical research question. The related Leontief-type model reads:

$$\boldsymbol{x = (I - A^x)^{-1} \cdot V^T \cdot \hat{q}^{-1} \cdot y} \tag{14}$$

Alternatively, this can be solved as:

$$\boldsymbol{x = V^T \cdot \hat{q}^{-1} \cdot (I - A^q)^{-1} \cdot y} \tag{15}$$

Both Leontief-type models can be extended by the environmental extension leading to:

$$\boldsymbol{R^x = \hat{s}^x \cdot (I - A^x)^{-1} \cdot V^T \cdot \hat{q}^{-1} \cdot y} \tag{16}$$

$$\boldsymbol{R^x = \hat{s}^x \cdot V^T \cdot \hat{q}^{-1} \cdot (I - A^q)^{-1} \cdot y} \tag{17}$$

For both cases, each element $\boldsymbol{R^x}_{ij}$ of the resulting physical industry by product matrix shows, how much of an environmental extension is used by industry *i* to produce intermediate goods for the production of final demand of product *j*. The row totals show the direct use of an environmental extension by industries *i*, that is the original environmental extension vector by industries ($\boldsymbol{r^x}$).

The column totals show the direct and indirect amount of an environmental extension activated by the final demand for product *j*.

Commodity-technology assumption (CTA)

The system of equations related to the CTA – two equations in two unknowns – can be represented in the following supply-use block (eq.[25] in Tukker and Heijungs 2008):

$$\begin{pmatrix} q \\ x \end{pmatrix} = \begin{pmatrix} 0 & U \cdot \hat{x}^{-1} \\ \hat{x} \cdot V^{-1} & 0 \end{pmatrix} \begin{pmatrix} q \\ x \end{pmatrix} + \begin{pmatrix} y \\ 0 \end{pmatrix} \tag{18}$$

As for ITA, the CTA leads to either a product or industry system. Solving the system for industry output gives the industry-by-industry requirement matrix and the related Leontief-type model is (note that the Eurostat manual terms this *fixed industry sales structure industry by industry model*; Eurostat 2008):

$$\boldsymbol{x} = (\boldsymbol{I}\text{-}\boldsymbol{A}^{x})^{-1} \cdot \boldsymbol{y}^{x} \tag{19}$$

where

$$\boldsymbol{A}^{x} = \hat{\boldsymbol{x}} \cdot \boldsymbol{V}^{-1} \cdot \boldsymbol{U} \cdot \hat{\boldsymbol{x}}^{-1} \tag{20}$$

and

$$\boldsymbol{y}^{x} = \hat{\boldsymbol{x}} \cdot \boldsymbol{V}^{-1} \cdot \boldsymbol{y} \tag{21}$$

As previously, the vector of environmental intensities is added to the model equation leading to:

$$\boldsymbol{R}^{x} = \hat{\boldsymbol{s}}^{x} \cdot (\boldsymbol{I} - \boldsymbol{A}^{x})^{-1} \cdot \hat{\boldsymbol{y}}^{x} \tag{22}$$

Also for the CTA case, each element R^{x}_{ij} of this resulting physical industry by industry matrix shows, how much of the environmental extension is used by industry *i* to produce intermediate goods for industry *j* in order to enable the latter to produce its goods for final demand. The row totals show the direct use of an environmental extension by industries *i*, i.e. the original environmental extension vector by industries ($\boldsymbol{r}^{x}$).

The column totals show the direct and indirect amount of environmental extension activated by the final demand of industry outputs *j*.

Solving the above CTA system for product outputs gives the product-by-product requirements matrix, the central block of a product-by-products IOT. The related Leontief-type model is:

$$q = (I-A^q)^{-1} \cdot y \tag{23}$$

where

$$A^q = U \cdot V^{-1} \tag{24}$$

The environmental extension is integrated by adding the vector of environmental intensities to the model equation; note, this time the intensities-vector is by products, i.e. s^q:

$$R^q = \hat{s}^q \cdot (I - A^q)^{-1} \cdot \hat{y} \tag{25}$$

Also for the CTA case, each element R^q_{ij} of this product by product matrix shows, how much of the environmental extension is directly used for the production of product *i* to be delivered as intermediate good for the production of final demand of product *j*. The row totals show the direct amount of a environmental extension needed for total output of product *i*, that is the environmental extension vector broken down by products (r^q).

The column totals show the direct and indirect amount of an environmental extension activated by the final demand of product *j*.

Also in the CTA case, a mixed system can be derived that takes the final demand in products and returns the industry output. The related Leontief-type model reads:

$$x = (I - A^x)^{-1} \cdot \hat{x} \cdot V^{-1} \cdot y \tag{26}$$

The environmental extension is added to the model resulting in a physical matrix as:

$$R^x = \hat{s}^x \cdot (I - A^x)^{-1} \cdot \hat{x} \cdot V^{-1} \cdot y \tag{27}$$

Each element R^x_{ij} of the resulting physical industry by product matrix shows, how much of an environmental extension is used by industry *i* to produce intermediate goods for the production of final demand of product *j*. The row totals show the direct amount of an environmental extension by industries *i*, that is the original environmental extension vector by industries (r^x).

The column totals show the direct and indirect amount of an environmental extension activated by the final demand for products *j*.

List of references

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Annex I: A numerical example

This section presents a numerical example to illustrate the mathematics of the different EE-SUT models as theoretically introduced in the previous sections.

The EE-SUT scheme as shown in Figure 2 shall be represented with the following numbers:

	products			industries					
products				5	15	5	10	5	40
				15	15	20	80	50	180
				5	25	45	100	55	230
industries	30	5	0						35
	10	175	0						185
	0	0	230						230
				10	130	160			
	40	180	230	35	185	230			
				30	530	230	5	0	

The vector of environmental intensities by industries is calculated using equation (3), leading to:

$$s^{xT} = r^{xT} \cdot \langle x \rangle^{-1}$$

0.9	2.9	1.0

The vector of environmental intensities by products can be derived with equation (4):

$$s^{q} = V^{-T} \cdot r^{x}$$

(4)

0.5
3
1.0

And the environmental factor input vector by products is obtained backwards by applying equation (5):

$r^{qT} = q^T \cdot \langle s^q \rangle$

20	540	230

Industry-technology assumption (ITA)

The industry-by-industry Leontief-type model under the ITA-assumption is as follows:

$x = (I-A^x)^{-1} \cdot y^x$ (7)

35
185
230

$A^x = V^T \cdot \langle q \rangle^{-1} \cdot U \cdot \langle x \rangle^{-1}$ (8)

0.1	0.1	0.0
0.5	0.1	0.1
0.1	0.1	0.2

$y^x = V^T \cdot \langle q \rangle^{-1}$ (9)

15
130
155

Integrating the environmental extension leads to the following physical matrix:

$R^x = \langle s^x \rangle \cdot (I-A^x)^{-1} \cdot \langle y^x \rangle$ (10)

15	9.9	5	30
27	438	65	530
4.7	28	198	230
47	476	267	790

The product-by-product model under the ITA is:

$q = (I-A^q)^{-1} \cdot y$ (11)

40
180
230

$A^q = U \cdot \langle x \rangle^{-1} \cdot V^T \cdot \langle q \rangle^{-1}$ (12)

0.1	0.1	0.0
0.3	0.1	0.1
0.1	0.1	0.2

Extending this model leads to the physical matrix:

$R^q = \langle s^q \rangle \cdot (I - A^q)^{-1} \cdot \langle y \rangle$

(13)

9	7.5	3.5	20
22	454	65	540
4.4	28	198	230
35	490	266	*790*

The two mixed industry-by-product model under the ITA assumption are:

$x = (I - A^x)^{-1} \cdot V^T \cdot \langle q \rangle^{-1} \cdot y$

(14)

35
185
230

$x = V^T \cdot \langle q \rangle - 1 \cdot (I - A^q)^{-1} \cdot y$

(15)

35
185
230

Both models can be extended leading to physical matrices like the following:

$R^x = \langle s^x \rangle \cdot (I - A^x)^{-1} \cdot V^T \cdot \langle q \rangle^{-1} \cdot \langle y \rangle$

(16)

12	13	5	30
33	432	65	530
4.4	28	198	230
49	474	267	*790*

$R^x = \langle s^x \rangle \cdot V^T \cdot \langle q \rangle^{-1} \cdot (I - A^q)^{-1} \cdot \langle y \rangle$

(17)

12	13	5	30
33	432	65	530
4.4	28	198	230
49	474	267	*790*

Industry-technology assumption (ITA)

The industry-by-industry Leontief-type model under the CTA-assumption is obtained by equations (19 to 21), as follows:

$$x = (I-A^x)^{-1} \cdot y^x$$

(19)

35
185
230

$$A^x = \langle x \rangle V^{-1} \cdot U \langle x \rangle^{-1}$$

(20)

0.1	0.1	0.0
0.4	0.1	0.1
0.1	0.1	0.2

$$y^x = \langle x \rangle V^{-1} \cdot$$

(21)

8.9
136
155

The corresponding environmentally extended model yields the following physical matrix:

$$R^x = \langle s^x \rangle \cdot (I-A^x)^{-1} \cdot \langle y^x \rangle$$

(22)

9.4	15	5.9	30
15	451	63	530
2.9	30	198	230
28	495	267	790

The product-by-product model under the CTA-assumption looks like:

$$q = (I-A^q)^{-1} \cdot y$$

(23)

40
180
230

$$A^q = U \cdot V^{-1}$$

(24)

0.2	0.1	0.0
0.5	0.1	0.1
0.1	0.1	0.2

The extended version reveals the following physical matrix:

$$R^q = \langle s^q \rangle \cdot (I-A^q)^{-1} \cdot \langle y \rangle$$

(25)

9.4	7.1	3.5	20
31	444	66	540
5.1	27	198	230
45	478	267	790

Finally, the mixed industry-by-product Leontief-model under the CTA-assumption yields the following:

$$x = (I-A^x)^{-1} \cdot \langle x \rangle V^{-1} \cdot y$$

(26)

35
185
230

The related environmentally extended model results into the following physical matrix:

$$R^x = \langle s^x \rangle \cdot (I-A^x)^{-1} \cdot \langle x \rangle \cdot V^{-1} \cdot \langle y \rangle$$

(27)

18	5.7	5.9	30
22	445	63	530
5.1	27	198	230
45	478	267	790

The following overview presents the six different results for the physical models, here only the vector of column-totals is compared as this is the most interesting outcome as it re-attributes the initial environmental extension vectors to the final demand:

Comparative overview of results for EE re-attributed to final demand

	ITA	CTA
industry-by-industry	47 476 267	28 495 267
product-by product	35 490 266	45 478 267
industry-by-product	49 474 267	45 478 267

The re-attribution model results differ quite substantially between ITA and CTA for two first models (industry-by-industry and product-by-product).

For the mixed model (industry-by-product) the differences between ITA and CTA are less pronounced, actually, the outcomes are quite close.

Annex II: Contributors to the report

This report is the result of discussions between all partners in the EXIOPOL consortium. It has been edited by Stephan Moll. The different chapters were written by the following persons:

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